IOSR Journal of Electronics and Communication Engineering (IOSR-JECE)
e-ISSN: 2278-2834,p- ISSN: 2278-8735.Volume 13, Issue 5, Ver. I (Sep.-Oct. 2018), PP 32-41
www.iosrjournals.org

Non-Uniform Lossless Transmission Lines Terminated ByElementWith Exponential Nonlinearity

Vasil Angelov
(Department of Mathematics, University of Mining and Geology "St.I.Rilski" − Sofia, Bulgaria)
Corresponding Author:Vasil Angelov

Abstract:*The present paper is devoted to the investigation of lossless transmission lines with varying in time and space specific parameters terminated by conductive loads with exponential nonlinearity. Such systems are more complicated because the coefficients of the hyperbolic system describing the line depend on time and space. We introduce new conditions which guarantee distortionless propagation along the transmission line and present a general method for reducing the mixed problem for the hyperbolic system to an initial value problem for a neutral system on the boundary. Here we extend our previous methods to investigate the case of variable specific parameters and formulate sufficient conditions for the existence-uniqueness of a solution by fixed point method.*

Date of Submission: 02-10-2018 Date of acceptance: 19-10-2018

I. Introduction

Hereweconsider non-uniform lossless transmission lines terminated by nonlinear conductive element with exponential type characteristic. Such a configuration has been studied in previous papers, but in the case of uniform transmission lines[1-13]. The main difficulty is caused by the varying in space and time specific parameters. That is why, the primary purpose of the present paper is to extend the author's method[10] for analysis oftransmission lines to transmission lines with varying per-unit-length specific parameters, that is, $C = C(x,t),\ L = L(x,t)$.Let us note that the particular case $C = C(x,t),\ L = \text{const}$ is applied to parametric amplification of traveling waves[14-17]. We obtain this case as a consequence of our theory.

Referring to Figure no 1, a non-uniform transmission line is shown terminated by a nonlinear conductive load with characteristic of exponentialtype $i = f(u) = I_{sat}\left(e^{\alpha u} - 1\right)$and parallel connected shuntcapacitance C_1, where $E(t)$ is the source function, R_0 −the source resistance, and Λ − the length of the line. The positiveconstants I_{sat}, α are explained inthe numerical example.

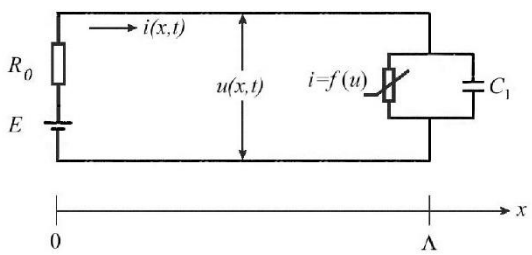

Figure. 1 Non-uniform lossless transmission line terminated by nonlinear conductive element

We consider a non-uniform lossless transmission line described by the system

$$\frac{\partial i(x,t)}{\partial x} = -\frac{\partial\left[C(x,t)\,u(x,t)\right]}{\partial t}\ ,\quad \frac{\partial u(x,t)}{\partial x} = -\frac{\partial\left[L(x,t)\,i(x,t)\right]}{\partial t} \qquad (1)$$

Extending results[14-17]we introduce conditions

(LC): $C(x,t) = C_0(1+\gamma\, c(x,t)),\ \left|c(x,t)\right| \le 1, 0 < \gamma < 1;\ L(x,t) = L_0(1+\gamma\, l(x,t)),\ \left|l(x,t)\right| \le 1, 0 < \gamma < 1.$

Then obviously $0 < C_0(1-\gamma) \le C(x,t) \le C_0(1+\gamma)$; $0 < L_0(1-\gamma) \le L(x,t) \le L_0(1+\gamma)$. Besides we assume that $\left|C_t(x,t)\right| \le C_{t0}$, $\left|C_x(x,t)\right| \le C_{x0}$, $\left|L_t(x,t)\right| \le L_{t0}$, $\left|L_x(x,t)\right| \le L_{x0}$, where $C_t = \partial C/\partial t;\ L_t = \partial L/\partial t$. Then the system (1) becomes

$$\frac{\partial u(x,t)}{\partial t}+\frac{1}{C(x,t)}\frac{\partial i(x,t)}{\partial x}+\frac{C_t(x,t)}{C(x,t)}u(x,t)=0,\ \frac{\partial i(x,t)}{\partial t}+\frac{1}{L(x,t)}\frac{\partial u(x,t)}{\partial x}+\frac{L_t(x,t)}{L(x,t)}i(x,t)=0. \qquad (2)$$

We formulate a mixed problem for system (2): to find unknown voltage $u(x,t)$ and current $i(x,t)$ satisfying (2) for $(x,t)\in\Pi=\{(x,t)\in R^2:(x,t)\in[0,\Lambda]\times[0,T]\}$, with boundary conditions (derived from Figure no 1 on the base of Kirchhoff's law)

$$E(t)-u(0,t)=R_0 i(0,t),\ t\ge 0;\ C_1\frac{du(\Lambda,t)}{dt}=i(\Lambda,t)-I_{sat}\left(e^{\alpha u(\Lambda,t)}-1\right),\ t\ge 0$$

and initial conditions

$$u(x,0)=u_0(x), i(x,0)=i_0(x)\ \ x\in[0,\Lambda],$$

where $u_0(x), i_0(x)$ are prescribed functions.

II. Methods

In this paper at first we use the method of reducing the mixed problem for a hyperbolic system to an initial value problem for neutral system on the boundary. The problem of existing of thesolution of the obtained neutral system we present as a fixed point problem for a suitable operator. By the fixed point method we establish existence-uniqueness of a solution.

III. Results

3.1. Transformation of the hyperbolic system in a diagonal form. Conditions for distortionless propagation.

The system (2)can be rewritten in matrix form

$$\frac{\partial U}{\partial t}+A\frac{\partial U}{\partial x}+A_1U=\bar{0}, \qquad (3)$$

where $U=\begin{bmatrix}u(x,t)\\ i(x,t)\end{bmatrix}$, $A=\begin{bmatrix}0 & \frac{1}{C(x,t)}\\ \frac{1}{L(x,t)} & 0\end{bmatrix}$, $A_1=\begin{bmatrix}\frac{C_t(x,t)}{C(x,t)} & 0\\ 0 & \frac{L_t(x,t)}{L(x,t)}\end{bmatrix}$, $\bar{0}=\begin{bmatrix}0\\0\end{bmatrix}$.

To transform A in a diagonal form we solve the equation: $\begin{vmatrix}-\lambda & 1/C(x,t)\\ 1/L(x,t) & -\lambda\end{vmatrix}=0$ whose roots are $\lambda_1(x,t)=1/\sqrt{L(x,t)C(x,t)},\ \lambda_2(x,t)=-1/\sqrt{L(x,t)C(x,t)}$. Eigen-vectors are $\left(\xi_1^{(1)},\xi_2^{(1)}\right)=\left(\sqrt{C(x,t)},\sqrt{L(x,t)}\right)$, $\left(\xi_1^{(2)},\xi_2^{(2)}\right)=\left(-\sqrt{C(x,t)},\sqrt{L(x,t)}\right)$. Denote by H the matrix formed by eigen-vectors: $H(x,t)=\begin{bmatrix}\sqrt{C(x,t)} & \sqrt{L(x,t)}\\ -\sqrt{C(x,t)} & \sqrt{L(x,t)}\end{bmatrix}$. Its inverse one is $H^{-1}(x,t)=\begin{bmatrix}1/\left(2\sqrt{C(x,t)}\right) & -1/\left(2\sqrt{C(x,t)}\right)\\ 1/\left(2\sqrt{L(x,t)}\right) & 1/\left(2\sqrt{L(x,t)}\right)\end{bmatrix}$ and

$$HAH^{-1}=\begin{bmatrix}1/\sqrt{L(x,t)C(x,t)} & 0\\ 0 & -1/\sqrt{L(x,t)C(x,t)}\end{bmatrix}\equiv A^{\text{diag}}.$$

Introduce a new variable $Z=\begin{bmatrix}V(x,t)\\ I(x,t)\end{bmatrix}$, where $Z=HU$ and $U=H^{-1}Z$ Uor

$$\left|\begin{array}{l}V(x,t)=\sqrt{C(x,t)}\,u(x,t)+\sqrt{L(x,t)}\,i(x,t)\\ I(x,t)=-\sqrt{C(x,t)}\,u(x,t)+\sqrt{L(x,t)}\,i(x,t)\end{array}\right. \qquad \left|\begin{array}{l}u(x,t)=[V(x,t)-I(x,t)]/2\sqrt{C(x,t)}\\ i(x,t)=[V(x,t)+I(x,t)]/2\sqrt{L(x,t)}.\end{array}\right.$$

Substituting $U=H^{-1}Z$ in (3) we obtain

$$\frac{\partial\left(H^{-1}Z\right)}{\partial t}+A\frac{\partial\left(H^{-1}Z\right)}{\partial x}+A_1\left(H^{-1}Z\right)=0.$$

It follows

$$H^{-1}\frac{\partial Z}{\partial t}+AH^{-1}\frac{\partial Z}{\partial x}+\left(\frac{\partial H^{-1}}{\partial t}+A\frac{\partial H^{-1}}{\partial x}+A_1H^{-1}\right)Z=0.$$

After multiplication from the left by H we obtain

$$\frac{\partial Z}{\partial t}+A^{\mathrm{diag}}\frac{\partial Z}{\partial x}+\left(H\frac{\partial H^{-1}}{\partial t}+HA\frac{\partial H^{-1}}{\partial x}+HA_1H^{-1}\right)Z=0. \qquad (4)$$

In view of the denotation $v=1/\sqrt{LC}$ we have

$$\frac{\partial H^{-1}(x,t)}{\partial t}=\frac{1}{4}\begin{bmatrix}-C_t/C\sqrt{C} & C_t/C\sqrt{C}\\ -L_t/L\sqrt{L} & -L_t/L\sqrt{L}\end{bmatrix},\ \frac{\partial H^{-1}(x,t)}{\partial x}=\frac{1}{4}\begin{bmatrix}-C_x/C\sqrt{C} & C_x/C\sqrt{C}\\ -L_x/L\sqrt{L} & -L_x/L\sqrt{L}\end{bmatrix}$$

and therefore

$$H\frac{\partial H^{-1}}{\partial t}=\frac{1}{4}\begin{bmatrix}-L_t/L-C_t/C & -L_t/L+C_t/C\\ -L_t/L+C_t/C & -L_t/L-C_t/C\end{bmatrix},\ HA\frac{\partial H^{-1}}{\partial x}=\frac{v}{4}\begin{bmatrix}-(C_x/C)-(L_x/L) & (C_x/C)-(L_x/L)\\ (C_x/C)+(L_x/L) & -(C_x/C)+(L_x/L)\end{bmatrix},$$

$$HA_1H^{-1}=\frac{1}{2}\begin{bmatrix}(C_t/C)+(L_t/L) & -(C_t/C)+(L_t/L)\\ -(C_t/C)+(L_t/L) & (C_t/C)+(L_t/L)\end{bmatrix}.$$

Then

$$H\frac{\partial H^{-1}}{\partial t}+HA\frac{\partial H^{-1}}{\partial x}+HA_1H^{-1}=\frac{1}{4}\begin{bmatrix}(L_t/L)+(C_t/C)-(vL_x/L)-(vC_x/C) & (L_t/L)-(C_t/C)-(vL_x/L)+(vC_x/C)\\ (L_t/L)-(C_t/C)+(vL_x/L)+(vC_x/C) & (L_t/L)+(C_t/C)+(vL_x/L)-(vC_x/C)\end{bmatrix}.$$

We rewrite (4) in explicit form:

$$\begin{aligned}&\frac{\partial V(x,t)}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial V(x,t)}{\partial x}+\frac{1}{4}\left(\frac{L_t}{L}+\frac{C_t}{C}-\frac{vL_x}{L}-\frac{vC_x}{C}\right)V(x,t)+\frac{1}{4}\left(\frac{L_t}{L}-\frac{C_t}{C}-\frac{vL_x}{L}+\frac{vC_x}{C}\right)I(x,t)=0\\ &\frac{\partial I(x,t)}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial I(x,t)}{\partial x}+\frac{1}{4}\left(\frac{L_t}{L}-\frac{C_t}{C}+\frac{vL_x}{L}+\frac{vC_x}{C}\right)V(x,t)+\frac{1}{4}\left(\frac{L_t}{L}+\frac{C_t}{C}+\frac{vL_x}{L}-\frac{vC_x}{C}\right)I(x,t)=0\ .\end{aligned} \qquad (5)$$

To simplify (5) we introduce conditions and call them **DistortionlessConditions(DC)**:

(DC1) $(L_t/L)+(C_t/C)-(vL_x/L)-(vC_x/C)=0$; **(DC2)** $(L_t/L)-(C_t/C)-(vL_x/L)+(vC_x/C)=0$;

(DC3) $(L_t/L)-(C_t/C)+(vL_x/L)+(vC_x/C)=0$; **(DC4)** $(L_t/L)+(C_t/C)+(vL_x/L)-(vC_x/C)=0$.

Then (5) becomes

$$\frac{\partial V(x,t)}{\partial t}+v\frac{\partial V(x,t)}{\partial x}=0,\ \frac{\partial I(x,t)}{\partial t}-v\frac{\partial I(x,t)}{\partial x}=0.$$

The transformation formulas are

$$\left|\begin{aligned}&V(x,t)=\sqrt{C(x,t)}\,u(x,t)+\sqrt{L}\,i(x,t)\\ &I(x,t)=-\sqrt{C(x,t)}\,u(x,t)+\sqrt{L}\,i(x,t)\end{aligned}\right. \text{ and } \left|\begin{aligned}&u(x,t)=V(x,t)/2\sqrt{C(x,t)}-I(x,t)/2\sqrt{C(x,t)}\\ &i(x,t)=V(x,t)/2\sqrt{L(x,t)}+I(x,t)/2\sqrt{L(x,t)}\,.\end{aligned}\right.$$

3.2. Formulation of the mixed problem and solutions of equations for the characteristics of the hyperbolic system

Now we are able to formulate the mixed problem with respect to the new variables. To find a solution of the system

$$\frac{\partial V(x,t)}{\partial t}+\frac{1}{\sqrt{LC}}\frac{\partial V(x,t)}{\partial x}=0,\ \frac{\partial I(x,t)}{\partial t}-\frac{1}{\sqrt{LC}}\frac{\partial I(x,t)}{\partial x}=0 \qquad (6)$$

satisfying the initial conditions

$$\left|\begin{aligned}&V(x,0)=\sqrt{C(x,0)}\,u(x,0)+\sqrt{L(x,0)}\,i(x,0)=\sqrt{C(x,0)}\,u_0(x)+\sqrt{L(x,0)}\,i_0(x)\equiv V_0(x)\\ &I(x,0)=-\sqrt{C(x,0)}\,u(x,0)+\sqrt{L(x,0)}\,i(x,0)=-\sqrt{C(x,0)}\,u_0(x)+\sqrt{L(x,0)}\,i_0(x)\equiv I_0(x)\end{aligned}\right.$$

and boundary conditions

$$E(t)-[(V(0,t)-I(0,t))/2\sqrt{C(0,t)}]=R_0V(0,t)+I(0,t)]/(2\sqrt{L})$$

$$C_1\frac{d}{dt}\left(\frac{V(\Lambda,t)}{2\sqrt{C(\Lambda,t)}}-\frac{I(\Lambda,t)}{2\sqrt{C(\Lambda,t)}}\right)=\frac{V(\Lambda,t)}{2\sqrt{L(\Lambda,t)}}+\frac{I(\Lambda,t)}{2\sqrt{L(\Lambda,t)}}-I_{sat}\left(e^{\alpha\left(\frac{V(\Lambda,t)}{2\sqrt{C(\Lambda,t)}}-\frac{I(\Lambda,t)}{2\sqrt{C(\Lambda,t)}}\right)}-1\right),\ t\ge 0.$$

Prior to reduce the above mixed problem to an initial value problem on the boundary[12]weconsiderthe Cauchy problems for the characteristics of the hyperbolic system (8), namely

$$\frac{dx_V(\tau)}{d\tau}=\frac{1}{\sqrt{L(x_V(\tau),\tau)C(x_V(\tau),\tau)}},\ x_V(t)=x\,;\ \frac{dx_I(\tau)}{d\tau}=-\frac{1}{\sqrt{L(x_V(\tau),\tau)C(x_I(\tau),\tau)}},\ x_I(t)=x\quad \text{for}\quad (x,t)\in\Pi. \tag{7}$$

Here the characteristic functions $\lambda_V(\xi,\tau)=\frac{1}{\sqrt{L(x_V(\tau),\tau)C(\xi,\tau)}}>0$ $\lambda_I(\xi,\tau)=-\frac{1}{\sqrt{L(x_V(\tau),\tau)C(\xi,\tau)}}<0$ are continuous ones. We need the following

Lemma 1. If $L(x,t),\,C(x,t)$ are continuous functionsthenfor every $(x,t)\in\operatorname{int}\Pi$ everyinitial value problemfrom(7) has a unique smooth solution in Π .

Proof: Let us consider the set $M_X=\left\{x(.)\in C[0,T]:|x(\tau)-x|\le X_0e^{\mu\tau}\right\}$, endowed with a metric

$\rho(x,\bar{x})=\max\left\{e^{-\mu\tau}|x(\tau)-\bar{x}(\tau)|:\tau\in[0,T]\right\}$,

where $C[0,T]$ is the space of all continuous functions and $X_0>0,\mu>0$ are constants.

Define the operator by the formula $B(x)(\tau):=x+\int_t^\tau\frac{1}{\sqrt{L(x(s),s)C(x(s),s)}}ds.$

We show that $B(x)$ maps M_X ino itself. Indeed, for sufficiently large $\mu>0$ we have

$$|B(x)(\tau)-x|\le\int_t^\tau\frac{1}{\sqrt{L(x(s),s)C(x(s),s)}}ds\le\frac{1}{(1-\gamma)\sqrt{L_0C_0}}\int_t^\tau e^{\mu s}ds\le\frac{1}{(1-\gamma)\sqrt{L_0C_0}}\frac{e^{\mu\tau}}{\mu}\le X_0e^{\mu\tau}.$$

To show that B is contractive operator we get

$$|B(x)(\tau)-B(\bar{x})(\tau)|\le\int_t^\tau\left|\frac{1}{\sqrt{L(x(s),s)C(x(s),s)}}-\frac{1}{\sqrt{L(\bar{x}(s),s)C(\bar{x}(s),s)}}\right|ds$$

$$\le\frac{1}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\int_\tau^{T_0}\left(|L(x(s),s)-L(\bar{x}(s),s)||C(x(s),s)|+|L(\bar{x}(s),s)||C(x(s),s)-C(\bar{x}(s),s)|\right)ds$$

$$\le\frac{(1+\gamma)(L_{x0}C_0+L_0C_{x0})}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\int_t^\tau|x(s)-\bar{x}(s)|ds\le\frac{(1+\gamma)(L_{x0}C_0+L_0C_{x0})}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\frac{e^{\mu\tau}-e^{\mu t}}{\mu}\rho(x,\bar{x})$$

$$\le\frac{(1+\gamma)(L_{x0}C_0+L_0C_{x0})}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\frac{e^{\mu\tau}}{\mu}\rho(x,\bar{x})\le\frac{(1+\gamma)(L_{x0}C_0+L_0C_{x0})}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\frac{e^{\mu\tau}}{\mu}\rho(x,\bar{x})$$

which implies $\rho(B(x),B(\bar{x}))\le\frac{(1+\gamma)(L_{x0}C_0+L_0C_{x0})}{2(1-\gamma)^3\sqrt{L_0^{\,3}C_0^{\,3}}}\frac{1}{\mu}\rho(x,\bar{x})$.

Consequently B is contractive operator for sufficiently large $\mu>0$. The fixed point of B is a unique solution of the above Cauchy problem.For the second equation we proceed in a similar way defining the operator

$$B(x)(\tau):=x-\int_t^\tau\frac{1}{\sqrt{L(x(s),s)C(x(s),s)}}ds.$$

Lemma 1 is thus proved.

Further on the solution of the first equation we denote by $x_V(s)$, while of the second one − by $x_I(s)$.

3.3. Reducing the mixed problem to an initial value problem on the boundary

Denote the delays by $T_1(t)=\Lambda\sqrt{L(x_V(t),t)C(x_V(t),t)}$, $T_2(t)=\Lambda\sqrt{L(x_I(t),t)C(x_I(t),t)}$.

For their derivatives we obtain and assume

$$\dot{T}_1(t)=\Lambda\frac{L_x(x_V(t),t)\dot{x}_VC(x_V(t),t)+L_t(x_V(t),t)C(x_V(t),t)+L(x_V(t),t)C_x(x_V(t),t)\dot{x}_V+L(x_V(t),t)C_t(x_V(t),t)}{2\sqrt{L(x_V(t),t)C(x_V(t),t)}}\ne1$$

,

$$\dot{T}_2(t)=\Lambda\frac{L_x(x_I(t),t)\dot{x}_IC(x_I(t),t)+L_t(x_I(t),t)C(x_I(t),t)+L(x_I(t),t)C_x(x_I(t),t)\dot{x}_I+L(x_I(t),t)C_t(x_I(t),t)}{2\sqrt{L(x_I(t),t)C(x_I(t),t)}}\ne1.$$

Then integrating the equations (6) along the characteristics from the point $(0,t-T_1(t))$ on the left boundary to the point (Λ,t) on the right boundary we obtain

$$\int_{t-T_1(t)}^{t}\left(\frac{\partial V}{dx}\frac{dx_V}{ds}+\frac{\partial V}{\partial s}\right)ds=\int_{t-T_1(t)}^{t}\frac{dV(x_V(s),s)}{ds}ds=V(\Lambda,t)-V(0,t-T_1(t))=0\,.$$

In a similar way integrating from $(0,t)$ to $(\Lambda,t-T_2(t))$ we obtain

$$\int_{t-T_2(t)}^{t}\left(\frac{\partial I}{dx}\frac{dx_I}{ds}-\frac{\partial I}{\partial s}\right)ds=\int_{t-T_2(t)}^{t}\frac{dI(x_I(s),s)}{ds}ds=I(0,t)-I(\Lambda,t-T_2(t))=0\,.$$

It follows

$$V(\Lambda,t)=V(0,t-T_1(t)),\ I(0,t)=I(\Lambda,t-T_2(t)). \tag{8}$$

Then using the denotation $Z_0(x,t)=\sqrt{L(x,t)/C(x,t)}$ we obtain

$$V(0,t)=\frac{2E(t)\sqrt{L(0,t)C(0,t)}}{\sqrt{L(0,t)}+R_0\sqrt{C(0,t)}}+\frac{\sqrt{L(0,t)}-R_0\sqrt{C(0,t)}}{\sqrt{L(0,t)}+R_0\sqrt{C(0,t)}}I(0,t)\ ;$$

$$\frac{dI(\Lambda,t)}{dt}=\frac{dV(\Lambda,t)}{dt}$$

$$-\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}+\frac{1}{C_1Z_0(\Lambda,t)}\right)V(\Lambda,t)+\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}-\frac{1}{C_1Z_0(\Lambda,t)}\right)I(\Lambda,t)+\frac{2\sqrt{C(\Lambda,t)}\ I_{sat}}{C_1}\left(e^{\alpha\left(\frac{V(\Lambda,t)-I(\Lambda,t)}{2\sqrt{C(\Lambda,t)}}\right)}-1\right),\ t\ge 0\,.$$

Assume that the unknown functions are $V(0,t)=V(t)$ and $I(\Lambda,t)=I(t)$. Then in view of (8) we have

$$V(t)=\frac{2E(t)Z_0(0,t)}{Z_0(0,t)+R_0}+\frac{Z_0(0,t)-R_0}{Z_0(0,t)+R_0}I(t-T_2(t))\ ;$$

$$\frac{dI(t)}{dt}=\frac{dV(t-T_1(t))}{dt}(1-\dot{T}_1(t))-\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}+\frac{1}{C_1Z_0(\Lambda,t)}\right)V(t-T_1(t))+\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}-\frac{1}{C_1Z_0(\Lambda,t)}\right)I(t)$$

$$+\frac{2\sqrt{C(\Lambda,t)}\ I_{sat}}{C_1}\left(e^{\alpha\frac{V(t-T_1(t))-I(t)}{2\sqrt{C(\Lambda,t)}}}-1\right).$$

The system obtained consists of differential and functional equations with time dependent delays and it is a particular case of neutral type ones[11]. In order to define the correct initial value problem we have to prescribe the initial interval and an initial function with its derivative.

3.4. Existence-uniqueness of a continuous solution

Lemma 2. The following inequalities are valid:

$$T_{\min}=\Lambda L_0C_0(1-\gamma)\le T_1(t)=\Lambda\sqrt{L(x_V(t),t)C(x_V(t),t)}\le \Lambda L_0C_0(1+\gamma)=T_{\max},$$

$$T_{\min}=\Lambda L_0C_0(1-\gamma)\le T_2(t)=\Lambda\sqrt{L(x_I(t),t)C(x_I(t),t)}\le \Lambda L_0C_0(1+\gamma)=T_{\max}\,.$$

The proof is straightforward.

Here we prove an existence-uniqueness theorem for continuous solution of the following system:

$$V(t)=\frac{2E(t)\sqrt{L(0,t)}}{Z_0(0,t)+R_0}+\frac{Z_0(0,t)-R_0}{Z_0(0,t)+R_0}I(t-T_2(t))\equiv F_V(V,I)(t)\ \ ,t\in[0,T],$$

$$\begin{aligned}\frac{dI(t)}{dt}&=\frac{dV(t-T_1(t))}{dt}(1-\dot{T}_1(t))-\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}+\frac{1}{C_1Z_0(\Lambda,t)}\right)V(t-T_1(t))+\\&+\left(\frac{C_t(\Lambda,t)}{2C(\Lambda,t)}-\frac{1}{C_1Z_0(\Lambda,t)}\right)I(t)+\frac{2\sqrt{C(\Lambda,t)}\ I_{sat}}{C_1}\left(e^{\alpha\frac{V(t-T_1(t))-I(t)}{2\sqrt{C(\Lambda,t)}}}-1\right)\equiv F_I(V,I)(t),\ \ t\in[0,T],\end{aligned} \tag{9}$$

$$V(t)=V_0(t)\ ,\ I(t)=I_0(t),\ \ t\in[-T_{\max},0]$$

where $T_{\min}=\min\{T_1;T_2\}$, $T_1=\min\{t-T_1(t):t\in[0,T]\}$, $T_2=\min\{t-T_2(t):t\in[0,T]\}$ and $V_0(.),\ I_0(.)\in C[T_{\min},0]$.

Let us note that the initial functions $V_0(t),\ I_0(t)$ can be obtained translating the initial functions $u_0(x), i_0(x)$ along the characteristics of thehyperbolic system[11].

Introduce the sets

$$M_V = \{V(.) \in C[T_{\min}, T] : V(t) = V_0(t), t \in [T_{\min}, 0] \text{ and } |V(t)| \le V_0 e^{\mu t},\ t \in [0,T]\}$$
$$M_I = \{I(.) \in C[T_{\min}, T] :\ I(t) = I_0(t), t \in [T_{\min}, 0] \text{ and } |I(t)| \le I_0 e^{\mu t},\ t \in [0,T]\},$$

where T, V_0, I_0, μ are positive constants, with metrics

$$\rho(V, \bar{V}) = \max\{e^{-\mu t}|V(t) - \bar{V}(t)| : t \in [0,T]\},\ \rho(I, \bar{I}) = \max\{e^{-\mu t}|I(t) - \bar{I}(t)| : t \in [0,T]\}.$$

The set $M_V \times M_I$ turns out into a complete metric space[11] with respect to the metric

$$\rho((V,I),(\bar{V},\bar{I})) = \max\{\rho(V,\bar{V}), \rho(I,\bar{I})\}.$$

A map $B:(X,A) \to (X,A)$ is called contractive if $\rho(B(V,I), B(\bar{V},\bar{I})) \le \kappa\, \rho((V,I),(\bar{V},\bar{I}))$, $0 < \kappa < 1$.

We assume **(IN):** $E(0) = 0;\ V_0(-T_1(0)) = 0;\ V_0(0) = 0;\ I_0(-T_2(0)) = 0;\ I_0(0) = 0$.

Using (9) we define an operator $B = (B_V(V,I), B_I(V,I)) : M_V \times M_I \to M_V \times M_I$ by the formulas

$$B_V(V,I)(t) := \begin{cases} \dfrac{2E(t)\sqrt{L(0,t)}}{Z_0(0,t)+R_0} + \dfrac{Z_0(0,t)-R_0}{Z_0(0,t)+R_0} I(t - T_2(t)),\ \ t \in [0,T], \\ V_0(t),\ \ t \in [T_{\min}, 0] \end{cases}$$

$$B_I(V,I)(t) := \begin{cases} V(t - T_1(t)) - \displaystyle\int_0^t \left(\frac{C_t(\Lambda,s)}{2C(\Lambda,s)} + \frac{1}{C_1 Z_0(\Lambda,s)} \right) V(s - T_1(s)) ds + \\ + \displaystyle\int_0^t \left(\frac{C_t(\Lambda,s)}{2C(\Lambda,s)} - \frac{1}{C_1 Z_0(\Lambda,s)} \right) I(s) ds + \int_0^t \frac{2\sqrt{C(\Lambda,s)}\ I_{sat}}{C_1} \left(e^{\alpha \frac{V(s-T_1(s)) - I(s)}{2\sqrt{C(\Lambda,s)}}} - 1 \right) ds,\ \ t \in [0,T], \\ I_0(t),\ \ t \in [T_{\min}, 0] \end{cases},$$

where $V(s - T_1(s)) = \vec{V}_0(s),\ I(t - T_2(t)) = \vec{I}_0(t)$. Here $\vec{V}_0(t),\ \vec{I}_0(t)$ are translated to the right functions from $C[T_{\min}, T]$ on $[0,T]$.

Remark 1. We call the solution of (9)asolution of the operator equation $(V,I) = (B_V(V,I), B_I(V,I))$. In this way we avoid the complicated conformity condition

$$\frac{dI(0)}{dt} = \frac{dV(-T_1(0))}{dt}(1 - \dot{T}_1(0)) - \left(\frac{C_t(\Lambda,0)}{2C(\Lambda,0)} + \frac{1}{C_1 Z_0(\Lambda,0)} \right) V(-T_1(0))$$
$$+ \left(\frac{C_t(\Lambda,0)}{2C(\Lambda,0)} - \frac{1}{C_1 Z_0(\Lambda,0)} \right) I(0) + \frac{2\sqrt{C(\Lambda,0)}\ I_{sat}}{C_1} \left(e^{\alpha \frac{V(-T_1(0)) - I(0)}{2\sqrt{C(\Lambda,0)}}} - 1 \right),\ t \ge 0.$$

Conditions **(IN)**ensure the continuity of $B_V(V,I)(t)$ and $B_I(V,I)(t)$:

$$F_V(V,I)(0) = \frac{2E(0)\sqrt{L(0,t)}}{Z_0(0,t)+R_0} + \frac{Z_0(0,t)-R_0}{Z_0(0,t)+R_0} I(-T_2(0)) = V_0(0),\ B_I(V,I)(0) = I_0(0).$$

The following lemma is valid:

Lemma 3. Problem (9) has a solution $(V(.), I(.)) \in M_V \times M_I$ iff the operator B has a fixed point in $M_V \times M_I$, that is, $V = B_V(V,I); I = B_I(V,I)$.

Theorem 1. Let the following conditions be fulfilled:

1) **(IN)** $E(0) = 0,\ V_0(-T_1(0)) = 0,\ V_0(0) = 0,\ I_0(-T_2(0)) = 0,\ I_0(0) = 0,\ V_0(.), I_0(.) \in C[T_{\min}, 0]$, $|\vec{V}_0(t)| \le V_0 e^{\mu t},\ |\vec{I}_0(t)| \le I_0 e^{\mu t}, t \in [0,T]$;

2) $E(0) = 0,\ |E(t)| \le V_0 e^{\mu t},\ t \in [0,T]\ (V_0 = I_0)$;;

3) Assumptions **(CL)** and **(DC)** are valid;

4) $\dfrac{\alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}}{2\sqrt{C_0(1-\gamma)}} < 2$;

5) $V_0\sqrt{L_0(1+\gamma)} / \left(\sqrt{L_0/C_0}\sqrt{(1-\gamma)/(1+\gamma)} + R_0\right) + I_0 e^{-\mu T_{\min}} \le V_0$;

6) $V_0 e^{-\mu T_{\min}} + \frac{V_0 e^{-\mu T_{\min}} + I_0}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}} + \frac{4\sqrt{C_0(1+\gamma)}\ I_{sat}\alpha e^{\mu T}}{C_1\left(4\sqrt{C_0(1-\gamma)} - \alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}\right)}\right) \le I_0$;

7) $K_V = e^{-\mu T_{\min}}\left|\left(\sqrt{L_0/C_0}\sqrt{(1+\gamma)/(1-\gamma)} - R_0\right)\right| / \left(\sqrt{L_0/C_0}\sqrt{(1-\gamma)/(1+\gamma)} + R_0\right) < 1$;

8) $K_I = e^{-\mu T_{\min}} + \frac{1}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}} + \sqrt{\frac{1+\gamma}{1-\gamma}}\frac{\alpha e^2 I_{sat}}{C_1}\right)\left(e^{-\mu T_{\min}} + 1\right) < 1$.

Then there exists a unique continuous solution of (9), belonging to $M_V \times M_I$.

Proof: We show that $B_V : M_V \times M_I \to M_V$ and $B_I : M_V \times M_I \to M_I$.

Indeed, conditions **(IN)**that $\left(B_V(t), B_I(t)\right)$ are continuous functions on $[0,T]$.

We show that $\left|B_V(V,I)(t)\right| \le V_0 e^{\mu t}, t \in [0,T]$ and $\left|B_I(V,I)(t)\right| \le I_0 e^{\mu t}, t \in [0,T]$. Indeed,

$$\left|B_V(V,I)(t)\right| \le \left|I(t-T_2(t))\right| + 2\left|E(t)\right|\sqrt{L_0(1+\gamma)} / \left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}} + R_0\right) \le I_0 e^{\mu t} e^{-\mu T_{\min}} + 2V_0 e^{\mu t} / \left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}} + R_0\right)$$

$$\le e^{\mu t}\left(I_0 e^{-\mu T_{\min}} + V_0\sqrt{L_0(1+\gamma)} / \left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}} + R_0\right)\right) \le e^{\mu t} V_0.$$

In the next estimates we use the following inequalityfor $|u| < 2$

$$\left|e^u - 1\right| \le |u| + \frac{|u|^2}{2!} + \frac{|u|^3}{3!} + \frac{|u|^4}{4!} + \ldots \le |u|\left(1 + \frac{|u|}{2} + \frac{|u|^2}{2^2} + \frac{|u|^3}{2^3} + \ldots\right) \le |u|\frac{1}{1-(|u|/2)} = \frac{2|u|}{2-|u|}.$$

Then

$$\left|B_I(V,I)(t)\right| \le \left|\int_0^t F_I(V,I)(s)ds\right|$$

$$\le \left|\int_0^t \frac{dV(s-T_1(s))}{dt} d(s-T_1(s))\right| + \left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\int_0^t \left(V_0 e^{\mu(s-T_1(s))} + I_0 e^{\mu s}\right)ds$$

$$+ \frac{2\sqrt{C_0(1+\gamma)}\ I_{sat}}{C_1}\int_0^t \left|e^{\alpha\frac{V_0 e^{\mu(s-T_1(s))} + I_0 e^{\mu s}}{2\sqrt{C_0(1-\gamma)}}} - 1\right|ds$$

$$\le \left|V(t-T_1(t))\right| + \left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\left(V_0 e^{-\mu T_{\min}} + I_0\right)\int_0^t e^{\mu s}ds + \frac{2\sqrt{C_0(1+\gamma)}\ I_{sat}}{C_1}\frac{2\frac{\alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}}{2\sqrt{C_0(1-\gamma)}}}{2 - \frac{\alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}}{2\sqrt{C_0(1-\gamma)}}}\int_0^t e^{\mu s}ds$$

$$\le e^{\mu t}e^{-\mu T_{\min}}V_0 + \frac{e^{\mu t}-1}{\mu}\left(V_0 e^{-\mu T_{\min}} + I_0\right)\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}} + \frac{2\sqrt{C_0(1+\gamma)}\ I_{sat}}{C_1}\frac{2\sqrt{C_0(1-\gamma)}\frac{\alpha e^{\mu T}}{\sqrt{C_0(1-\gamma)}}}{4\sqrt{C_0(1-\gamma)} - \alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}}\right)$$

and therefore

$$\left|B_I(V,I)(t)\right| \le e^{\mu t}\left[V_0 e^{-\mu T_{\min}} + \frac{V_0 e^{-\mu T_{\min}} + I_0}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}} + \frac{4\sqrt{C_0(1+\gamma)}\ I_{sat}\alpha e^{\mu T}}{C_1\left(4\sqrt{C_0(1-\gamma)} - \alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}\right)}\right)\right]$$

$\le I_0 e^{\mu t}$.

It remains to show that the operator B is contractive one. Indeed, for the first component in view of the estimate from Lemma 2 we have

$$\left|B_V(V,I)(t) - B_V(\bar{V},\bar{I})(t)\right| \le \left|Z_0(0,t) - R_0\right| / \left|Z_0(0,t) + R_0\right|\left|I(t-T_2(t)) - \bar{I}(t-T_2(t))\right|$$

$$\le \left|Z_0(0,t) - R_0\right| / \left|Z_0(0,t) + R_0\right|\left|I(t-T_2(t)) - \bar{I}(t-T_2(t))\right| \le \left|Z_0(0,t) - R_0\right| / \left|Z_0(0,t) + R_0\right|\rho(I,\bar{I})e^{\mu(t-T_2(t))}$$

$$\le e^{\mu t}e^{-\mu T_{\min}}\rho(I,\bar{I})\left|\sqrt{\frac{L_0(1+\gamma)}{C_0(1-\gamma)}}-R_0\right|/\left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}}+R_0\right)\le e^{\mu t}e^{-\mu T_{\min}}\rho\left((V,I),(\bar{V},\bar{I})\right)\left|\sqrt{\frac{L_0(1+\gamma)}{C_0(1-\gamma)}}-R_0\right|/\left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}}+R_0\right).$$

It follows

$$\rho\left(B_V(V,I),B_V(\bar{V},\bar{I})\right)\le e^{-\mu T_{\min}}\rho\left((V,I),(\bar{V},\bar{I})\right)\left|\sqrt{\frac{L_0(1+\gamma)}{C_0(1-\gamma)}}-R_0\right|/\left(\sqrt{\frac{L_0(1-\gamma)}{C_0(1+\gamma)}}+R_0\right)\equiv K_V\rho\left((V,I),(\bar{V},\bar{I})\right).$$

For the second component we obtain

$$\left|B_I(V,I)(t)-B_I(\bar{V},\bar{I})(t)\right|\le\left|\int_0^t F_I(V,I)(s)ds-\int_0^t F_I(\bar{V},\bar{I})(s)ds\right|$$

$$\le\left|\int_0^t\left(\frac{dV(s-T_1(s))}{ds}-\frac{d\bar{V}(s-T_1(s))}{ds}\right)ds\right|$$

$$+\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\int_0^t\left|V(s-T_1(s))-\bar{V}(s-T_1(s))\right|ds+\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\int_0^t\left|I(s)-\bar{I}(s)\right|ds$$

$$+\frac{2\sqrt{C_0(1+\gamma)}\,I_{sat}}{C_1}\int_0^t\left|e^{\alpha\frac{V(t-T_1(t))-I(t)}{2\sqrt{C(\Lambda,t)}}}-e^{\alpha\frac{\bar{V}(t-T_1(t))-\bar{I}(t)}{2\sqrt{C(\Lambda,t)}}}\right|ds$$

$$\le\left|V(t-T_1(t))-\bar{V}(t-T_1(t))\right|+\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)e^{-\mu T_{\min}}\rho\left(V,\bar{V}\right)\frac{e^{\mu t}}{\mu}+\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\rho\left(I,\bar{I}\right)\frac{e^{\mu t}}{\mu}$$

$$+\frac{2\sqrt{C_0(1+\gamma)}\,I_{sat}}{C_1}\alpha\,e^{\alpha\frac{V_0e^{-\mu T_{\min}}+I_0}{2\sqrt{C_0(1-\gamma)}}e^{\mu T}}\int_0^t\frac{\left|V(s-T_1(s))-\bar{V}(s-T_1(s))\right|+\left|I(s)-\bar{I}(s)\right|}{2\sqrt{C_0(1-\gamma)}}ds$$

$$\le e^{\mu t}e^{-\mu T_{\min}}\rho(V,\bar{V})+\frac{e^{\mu t}}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)e^{-\mu T_{\min}}\rho\left(V,\bar{V}\right)+\frac{e^{\mu t}}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}\right)\rho\left(I,\bar{I}\right)$$

$$+\frac{2\sqrt{C_0(1+\gamma)}\,I_{sat}}{2\sqrt{C_0(1-\gamma)}\,C_1}\alpha\,e^{\alpha\frac{V_0e^{-\mu T_{\min}}+I_0}{2\sqrt{C_0(1-k)}}e^{\mu T}}\left(\rho(V,\bar{V})e^{-\mu T_{\min}}+\rho(I,\bar{I})\right)\frac{e^{\mu t}}{\mu}$$

$$\le e^{\mu t}\left[e^{-\mu T_{\min}}+\frac{1}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}+\frac{\sqrt{1+\gamma}\,I_{sat}}{\sqrt{1-\gamma}\,C_1}\alpha e^2\right)\left(e^{-\mu T_{\min}}+1\right)\right]\max\left\{\rho(V,\bar{V}),\rho\left(I,\bar{I}\right)\right\}.$$

It follows

$$\rho\left(B_I(V,I),B_I(\bar{V},\bar{I})\right)\le K_I\rho((V,I),(\bar{V},\bar{I})),$$

where

$$K_I=e^{-\mu T_{\min}}+\frac{e^{-\mu T_{\min}}+1}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)}+\frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}}+\frac{\sqrt{1+\gamma}\,I_{sat}\alpha e^2}{\sqrt{1-\gamma}\,C_1}\right)<1$$

and then

$$\rho\left(\left(B_V(V,I),B_I(V,I)\right),\left(B_V(\bar{V},\bar{I}),B_I(\bar{V},\bar{I})\right)\right)\le K\rho((V,I),(\bar{V},\bar{I})),$$

where $K=\max\{K_V,K_I\}$.

Therefore the operator *B* is contractive one and has a unique fixed point which is a solution of (9).
Theorem 1 is thus proved.

IV. Discussion

Here we consider the particular case of space-time depending specific parameters in the form: $L=\text{const}$, $C(x,t)=C_0\left(1+\gamma\cos(\omega_0 t-\beta x)\right)$, $C_t(x,t)=C_0\left(-\omega_0\gamma\sin(\omega_0 t-\beta x)\right)$ [11-13].We have obtained the following relation for the specific parameter $\frac{\partial C(x,t)}{\partial t}-\left(1/\sqrt{LC(x,t)}\right)\frac{\partial C(x,t)}{\partial x}=0$ .Then

$$\frac{dt}{1}=\frac{dx}{-1/\sqrt{LC(x,t)}}\Rightarrow\frac{dx}{dt}=-\frac{1}{\sqrt{LC_0\left(1+\gamma\cos(\omega_0 t-\beta x)\right)}}.$$

The last equation has a unique solution.

We take a nonlinear conductive elementwith exponential *V-I* characteristic $f(u) = I_{sat}\left(e^{\alpha u} - 1\right)$with an interval of negative differential resistance. Let us take $V_0 \approx I_0 \approx E_0 \approx 10^{-12}$, $\gamma = 0{,}01$, $\sqrt{\frac{1+\gamma}{1-\gamma}} \approx 1$, $\Lambda = 1m$,

$L = 0{,}2\,\mu H/m,\ C_0 = 5\,pF/m,\ R_0 = 35\Omega,\ C_1 = 10^{-10}F$; $\alpha = q/(\eta KT) = 37$ .Then

$\sqrt{LC_0} = \sqrt{\left(0{,}2.10^{-6}\right)\left(5.10^{-12}\right)} = 10^{-9}$; $\quad \sqrt{L/C_0} = \sqrt{\left(0{,}2.10^{-6}\right)/\left(5.10^{-12}\right)} = \sqrt{0{,}04.10^{6}} = 200$; $T = 10^{-8}$;

$T_{\min} = \Lambda\sqrt{LC_0(1-\gamma)} \le T(t) = \Lambda\sqrt{LC(x,t)} \le \Lambda\sqrt{LC_0(1+\gamma)} = T_{\max}$;

$(0{,}99)\times 10^{-9} \le T(t) = \Lambda\sqrt{LC(x,t)} \le (1{,}01)\times 10^{-9}$.

Let us take $\mu = 10^9$. Then $\mu T_{\min} = 10^9 \times 0{,}99 \times 10^{-9} = 0{,}99 \Rightarrow e^{\mu T_{\min}} = e^{0{,}99} \Rightarrow\ e^{-0{,}99} \approx 0{,}37$;

$\mu T = \left(10^9\right)\times\left(10^{-8}\right) = 10$; $e^{\mu T} = e^{10} = 22026; C_t(x,t) = -C_0\gamma\omega_0 \sin(\omega t - \beta x) \Rightarrow C_{t0} \le C_0\gamma\omega_0$.

Further on the inequalities from Theorem 1 become

$$\frac{\alpha\left(e^{-\mu T_{\min}}+1\right)e^{\mu T}}{2\sqrt{C_0(1-\gamma)}}V_0 < 2\ ;\ \sqrt{L_0(1+\gamma)}\Big/\left(\sqrt{\frac{L}{C_0(1+\gamma)}} + R_0\right) + e^{-\mu T_{\min}} \le 1;$$

$$e^{-\mu T_{\min}} + \frac{e^{-\mu T_{\min}}+1}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L}} + \frac{4\sqrt{C_0(1+\gamma)}\ I_{sat}\alpha e^{\mu T}}{C_1\left(4\sqrt{C_0(1-\gamma)} - \alpha\left(V_0 e^{-\mu T_{\min}} + I_0\right)e^{\mu T}\right)}\right) \le 1;$$

$$K_V = e^{-\mu T_{\min}}\left|\sqrt{\frac{L}{C_0(1-\gamma)}} - R_0\right| \Big/ \left(\sqrt{\frac{L}{C_0(1+\gamma)}} + R_0\right) < 1;$$

$$K_I = e^{-\mu T_{\min}} + \frac{e^{-\mu T_{\min}}+1}{\mu}\left(\frac{C_{t0}}{2C_0(1-\gamma)} + \frac{1}{C_1}\sqrt{\frac{C_0(1+\gamma)}{L_0(1-\gamma)}} + \frac{\sqrt{1+\gamma}\ I_{sat}\alpha\, e^2}{\sqrt{1-\gamma}\, C_1}\right) < 1$$

or

$$\frac{37\left(e^{-\mu T_{\min}}+1\right)e^{\mu T}}{2\sqrt{C_0(1-\gamma)}}V_0 < 2 \Leftrightarrow \frac{51{,}8}{4{,}45}\frac{e^{10}}{10^{-6}}10^{-12} < 2 \Leftrightarrow\ 11{,}64.10^{-6}.22026 < 2\ ;\ 10^{-3}.0{,}003 + 0{,}4 \le 1\ ;$$

$$0{,}4\,0{,}4 + \frac{1{,}4}{10^9}\left[\frac{0{,}01\omega_0}{2.0{,}99} + \frac{1}{200.8.10^{-11}} + \frac{4.10^{-8}.10^{-6}\sqrt{5}}{8.10^{-11}}\frac{37.22026}{4.10^{-6}\sqrt{5} - 51{,}8.10^{-12}.22026}\right] \le 1\ ;$$

$$K_V = 0{,}4\frac{|200-35|}{235} = 0{,}28 < 1\ ;\ K_I = 0{,}4 + \frac{1{,}4}{10^9}\left(\frac{0{,}01\omega_0}{2.0{,}99} + \frac{1}{200.8.10^{-11}} + \frac{37.10^{-8}}{8.10^{-11}}e^2\right) < 1.$$

Finally we have to find an interval for the constant $\omega_0 : \omega_0 \le \frac{(0{,}6 - 0{,}0625 - 0{,}323).10^9}{0{,}007} \approx 3.10^{10}$

$\omega_0 \le 7{,}75.10^{13}$ and $\omega_0 \le 7.10^{13}$ .Then

$$K_I = 0{,}4 + \frac{1}{10^9}\left(0{,}0099\omega_0 + 0{,}088.10^9 + 48.10^3\right) < 1 \Leftrightarrow \omega_0 < \frac{4{,}12 - 48.10^{-5}}{0{,}0099}.10^8 \approx 4{,}16.10^{10}.$$

Therefore for $\omega_0 = 10^{10}$ we obtain $K_I = 0{,}4 + 0{,}099 + 0{,}088 + 48.10^{-6} \approx 0{,}4 + 0{,}099 + 0{,}088 = 0{,}587 < 1$.

V. Conclusion

Applying our methods, we have found explicitconditions for the specific parameters in order to obtain distortionless signal. Moreover, we can obtain an approximated but explicit solution for voltage and current. We can choose an initial approximation $V(t) = V_0 \sin\theta(t), I(t) = I_0 \cos\theta(t)$.

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IOSR Journal of Electronics and Communication Engineering (IOSR-JECE) is UGC approved Journal with Sl. No. 5016, Journal no. 49082.

Vasil Angelov. "Non-Uniform Lossless Transmission Lines Terminated By Element With Exponential Nonlinearity." IOSR Journal of Electronics and Communication Engineering (IOSR-JECE) 13.5 (2018): 32-41.